MODELLING IN MECHANICS

KEY WORDS & DEFINITIONS

1. **Model** – A mathematical system which enables a problem to be solved
2. **Light** – Has negligible mass
3. **Static** – Not moving
4. **Rigid** – Doesn't bend
5. **Thin** – Has negligible thickness
6. **Smooth** – Has a surface that results in no friction between itself and an object
7. **Rough** – Has a surface that requires frictional forces between itself and an object to be considered
8. **Particle** – Dimensions are negligible, so mass or object is at a point. Rotational forces and air resistance can be ignored.
9. **Rod** – A long, thin, straight object. Mass is along a line that is rigid.
10. **Lamina** – A thin 2-dimensional surface with mass distributed evenly across its flat surface.
11. **Uniform Body** – Mass is distributed evenly, so acts at the centre of mass.
12. **Light string** – Has negligible mass and equal tension at both ends.
13. **Inextensible string** – A string that does not stretch so that connected objects can move with the same acceleration if the string is taut.
14. **Wire** – A rigid, thin length of metal.
15. **Smooth and Light Pulley** – A pulley that has no mass and results in tension being equal on either side.
16. **Bead** – A particle with a hole in it which can freely move along a wire or string, resulting in equal tension either side of the bead.
17. **Peg** – A supporting object that is dimensionless and fixed but may be rough or smooth.
18. **Air Resistance** – The resistance force as experienced as on object moves through the air, which is often modelled as negligible.
19. **Gravity** – The force of attraction between objects.
20. **Earth's Gravity** – Assumed to apply to all objects with mass. Acts uniformly and vertically downwards with a value of $9.8 m/s^2$
21. **Scalar** – A quantity which has magnitude only – distance, speed, time, mass. Always positive.
22. **Vector** – A quantity which has magnitude and direction – displacement, velocity, acceleration, force, weight. Can be described using column or i j notation. Can be positive or negative.

Distance is the magnitude of the displacement vector

Speed is the magnitude of the velocity vector

SI BASE UNITS

Quantity	Mass	Length/ Displacement	Time	Speed/ Velocity	Acceleration	Weight/ Force
Symbol	kg	m	s	ms^{-1}	ms^{-2}	N (= $kgms^{-2}$)

CONSTANT ACCELERATION

KEY WORDS & DEFINITIONS

1. **Velocity**
   The rate of change of displacement
2. **Acceleration**
   The rate of change of velocity

SUVAT EQUATIONS

For motion in a straight line with constant acceleration:

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

s – displacement
u – initial velocity
v – final velocity
a – acceleration
t - time

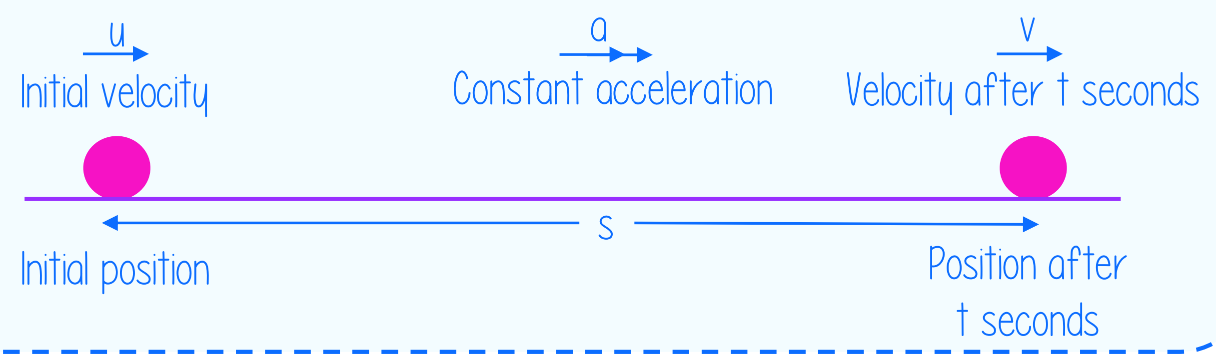
To derive the SUVAT equations:

- Find the gradient of a velocity time graph labelled with u, v, t
- Find the area underneath the velocity-time graph
- Use these two equations to replace each variable at a time to derive the other three equations.

WHAT DO I NEED TO KNOW

1. The gradient on a displacement-time graph = velocity
2. If a displacement-time graph is a straight line then the velocity is constant.
3. The gradient on a velocity-time graph = acceleration
4. If a velocity-time graph is a straight line then the acceleration is constant.
5. The area between a velocity-time graph and the time axis = Distance travelled
6. Average Speed = $\frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$
7. Average velocity = $\frac{\text{Displacement From Start Point}}{\text{Total Time Taken}}$
8. Acceleration due to gravity = $9.8 m/s^2$
9. Acceleration due to gravity does not depend on the mass of the object.
10. The degree of accuracy in your answers must be consistent with the values given in the question. I.e. if $g = 10 m/s^2$ in the question, your answer should also be given to 1 sig. fig.

ALWAYS DRAW A SKETCH!



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Forces & Motion

Key Words & Definitions

1. Resultant Force

The result of resolving forces on an object in a particular direction.

2. Weight

The force due to gravity acting on an object

Newton's Laws of Motion

Newton's First Law of Motion

Objects in equilibrium will not accelerate. An object will only accelerate (or decelerate) if an unbalanced force acts on the object.

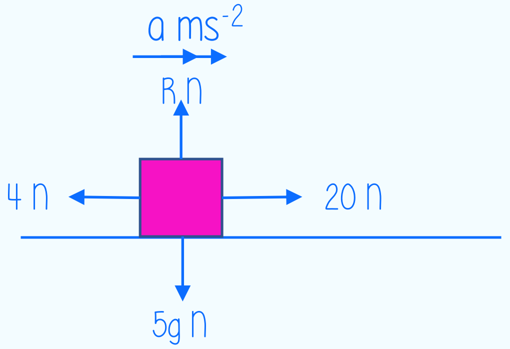
Newton's Second Law of Motion

The acceleration of an object depends on the overall net force acting on the object and the object's mass.

Newton's Third Law of Motion

For every action there is an equal and opposite reaction.

Always Draw a Diagram!



Formulae

Formula of Newton's Second Law of Motion

$$F = ma$$

Formula to calculate the weight of an object

$$W = mg$$

What Do I Need to Know

1. To resolve forces given as vectors add the vectors

   If 2 forces $(p\mathbf{i} + q\mathbf{j})$ N and $(r\mathbf{i} + s\mathbf{j})$ N are acting on a particle, the resultant force will b $((p + r)\mathbf{i} + (q + s)\mathbf{j})$ N

2. To solve problems involving connected particles moving in the same straight line consider the particles as a single unit, moving as one.

   Particles need to remain in contact or be connected by an inextensible rod or string to be considered a single particle

3. To solve problems involving connected particles that are not moving in the same straight line consider the particles, and the forces acting on them, separately.

   Particles need to be considered separately in order to find the tension in any string between them

4. The tension in an inextensible string passing over a smooth pulley is the same on both sides

   You cannot treat a system involving a pulley as a single particle as the particles are moving in different directions

VARIABLE ACCELERATION

KEY WORDS & DEFINITIONS

1. **Velocity**
   The rate of change of displacement

2. **Acceleration**
   The rate of change of velocity

FORMULAE

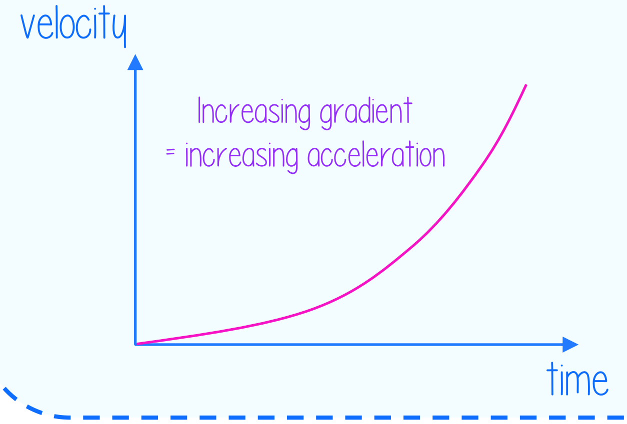
Velocity, if displacement is a function of time:

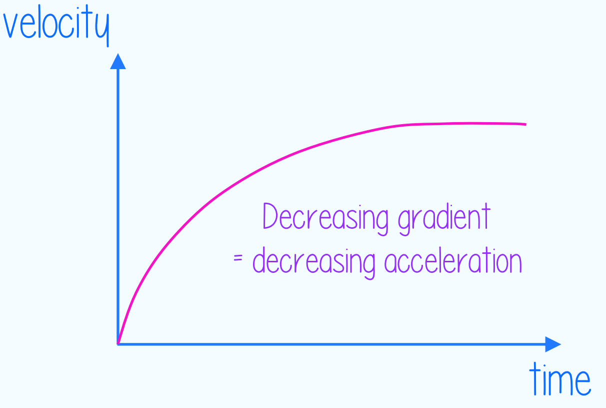
$$v = \frac{\mathrm{d}s}{\mathrm{d}t}$$

Acceleration, if velocity is a function of time

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2s}{\mathrm{d}t^2}$$

VELOCITY TIME GRAPHS





WHAT DO I NEED TO KNOW

1. The area under a velocity time graph represents the displacement
2. Integration is the reverse process to differentiation
3. Differentiate displacement with respect to time to get velocity
4. Differentiate velocity with respect to time to get acceleration
5. Integrate acceleration with respect to time to get velocity

$$\int (a)\,\mathrm{d}t = v$$

6. Integrate velocity with respect to time to get displacement

$$\int (v)\,\mathrm{d}t = s$$

7. The suvat equations can only be used when the acceleration is constant

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MOMENTS

KEY WORDS & DEFINITIONS

1. Moment
The turning effect of a force on a rigid body.

2. Resultant Moment
The sum of all moments acting on a rigid body.

3. The Point of Tilting
The instantaneous situation where the reaction at any support or the tension in any supporting string or wire, other than at the pivot, will be zero.

4. Coplanar Forces
Forces that act in the same plane.

5. Lamina
A 2D object whose thickness can be ignored.

MODELLING ASSUMPTIONS & IMPLICATIONS

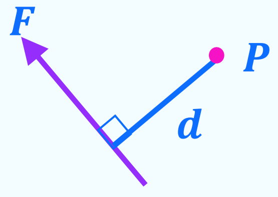
A plank is uniform ⇒ Weight acts at the centre of the plank

A plank is a rod ⇒ The plank remains straight

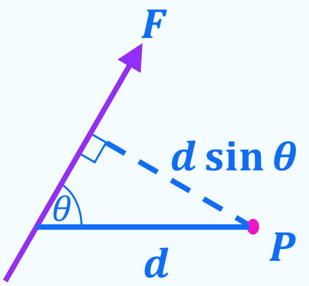
Any people/objects are particles ⇒ Their weight acts at the end of any rod

FORMULAE

Moment about P = magnitude of force x perpendicular distance of the force from P



Moment of **F** about P = | **F** | x d **N**m clockwise



Moment of **F** about P = FdSinθ **N**m clockwise

WHAT DO I NEED TO KNOW

1. The **units** of Moments are **Newton metres Nm**

2. The **direction** of the Moment (clockwise or anticlockwise) must be included with a moment's value.

3. When a rigid body is in **equilibrium**, the **resultant force** in any direction is 0**N** and the **resultant moment** about any point is 0**N**m

4. The centre of mass of a **non-uniform rod** is **not** necessarily at the **midpoint** of the rod.

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FORCES & FRICTION

KEY WORDS & DEFINITIONS

1. Friction
A force which opposes motion.

2. Coefficient of Friction μ
A measure of how resistant to motion two surfaces are

3. Limiting Equilibrium
The point at which there is equilibrium, but friction is at its maximum

FORMULAE

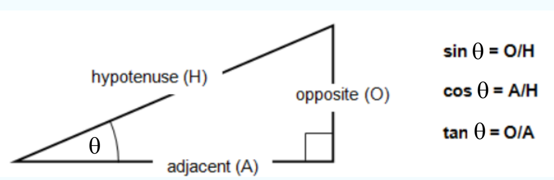
To calculate Maximum Friction:

$$F_{max} = \mu R$$

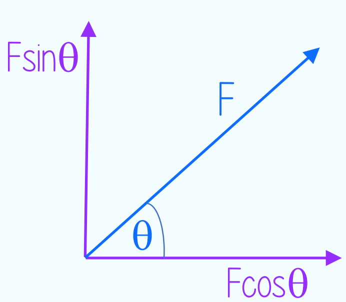
Where:
F is the frictional force
μ is the coefficient of friction
R is the normal reaction between the surfaces.

RESOLVING FORCES

"Cos through, Sine away"
Using SOH CAH TOA



when resolving forces gives the following result:



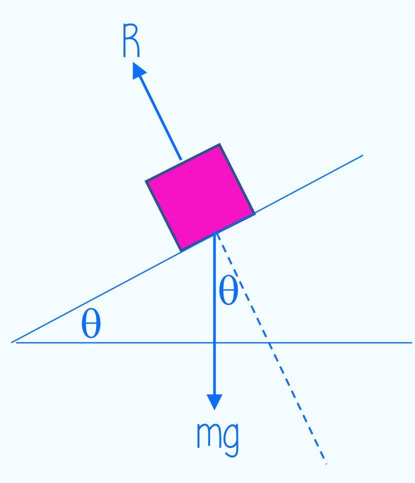
Component $F_x = F\cos\theta$ (through the angle)

Component $F_y = F\sin\theta$ (away from the angle)
(or $= F\cos(90 - \theta)$)

WHAT DO I NEED TO KNOW

1. If a force is applied at an angle to the direction of motion, resolve it in two perpendicular directions to find the component of force that acts in the direction of motion OR use the triangle law for vector addition.

2. To solve problems on inclined planes, resolve parallel and perpendicular to the plane. REMEMBER, the normal reaction force acts at right angles to the plane, not vertically.



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PROJECTILES

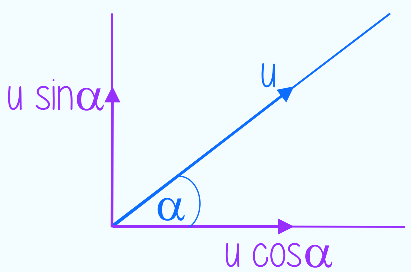
KEY WORDS & DEFINITIONS

1. **Projectile**
   A particle moving in a vertical plane under the action of gravity.
2. **Angle of Projection**
   The initial angle the projectile makes with the horizontal direction.
3. **Speed**
   The magnitude of the velocity, or the resultant velocities.
4. **Range**
   The horizontal distance that the particle travels.
5. **Time of Flight**
   The time taken for the projectile to hit the ground, or other horizontal surface, after being projected.

HORIZONTAL & VERTICAL COMPONENTS OF INITIAL VELOCITY

If a particle is projected with an initial velocity u, at an angle α above the horizontal, α is called 'The angle of projection'.

The velocity can be resolved into components that act horizontally and vertically.



The horizontal component of the initial velocity = u cos α

The vertical component of the initial velocity = u sin α

WHAT DO I NEED TO KNOW

1. The horizontal acceleration of a particle = 0
2. The horizontal velocity of a particle is constant. Therefore s = vt
3. The vertical acceleration a of a partical = g (constant)
4. To find the horizontal & vertical components of the initial velocity, resolve horizontally & vertically
5. When a projectile reaches its maximum height, the vertical component of velocity = 0
6. Acceleration due to gravity = 9.8m/s^2
   This does not depend on the mass of the object.
7. The degree of accuracy in your answers must be consistent with the values given in the question. I.e. if g = 10m/s^2 in the question, your answer should also be given to 1 sig. fig. Do not leave exact surd answers.
8. Many projectile problems can be solved by first using the vertical motion to find the total time taken.

POSSIBLE EQUATIONS TO DERIVE

For a particle projected with initial velocity U at angle α above horizontal and moving freely under gravity:

- Time of flight $= \dfrac{2U \sin \alpha}{g}$
- Time to reach greatest height $= \dfrac{U \sin \alpha}{g}$
- Range on horizontal plane $= \dfrac{U^2 \sin 2\alpha}{g}$
- Equation of trajectory:

$$y = x \tan \alpha - \frac{gx^2}{2U^2}(1 + \tan^2 \alpha)$$

where y is the vertical height of particle and x is the horizontal distance from the point of projection.

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APPLICATIONS OF FORCES

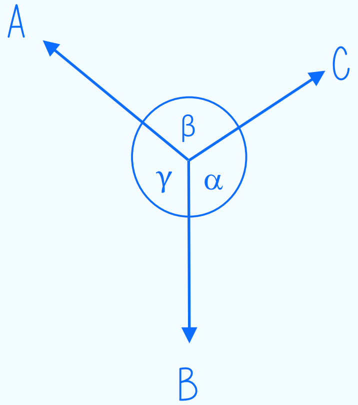
KEY WORDS & DEFINITIONS

Static Equilibrium

A particle is in static equilibrium if it is at rest and the resultant force acting upon it = 0

A rigid body is in static equilibrium if the body is stationary, the resultant force in any direction = 0 and the resultant moment = 0

LAMI'S THEOREM



$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

MODELLING

If a particle is attached separately to two strings, the tension can be different in each string.

If a smooth bead is threaded on a string, the tension in the string will be the same on both sides.

Unless connected particles are moving in the same direction, they must be considered separately.

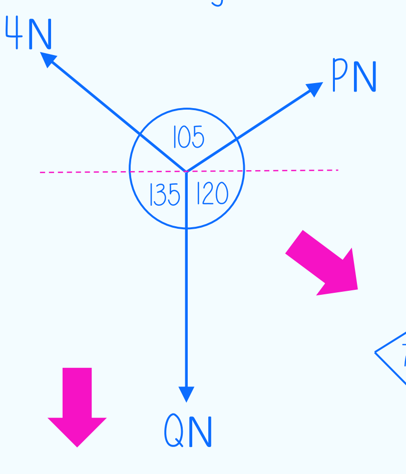
WHAT DO I NEED TO KNOW

1. The maximum value of the frictional force $F_{max} = \mu R$ is reached when the body being considered is on the point of moving. The body is then said to be in 'limiting equilibrium'.

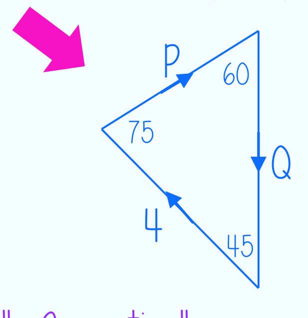
2. In general, the force of friction F is such that $F \leq \mu R$ and the direction of the frictional force is opposite to the direction in which the body would move, if the frictional force were absent.

3. To solve equilibrium problems, draw both a force diagram and a vector diagram.

4. If the angle between forces on a force diagram is θ, the angle between those forces in a triangle of forces is $180° - \theta$. The length of each side of the triangle is the magnitude of the force. (If the particle is not in equilibrium, the vector diagram will not be a closed triangle).



Method 2:
Use Sine Rule



Method 1:
Resolve horizontally & vertically

→ $P\cos 30 - 4\cos 45 = 0$

↑ $P\sin 30 + 4\sin 45 - Q = 0$

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FURTHER KINEMATICS

WHAT DO I NEED TO KNOW

1. To solve problems involving constant acceleration in 2 dimensions, use the SUVAT equations with vector components where
u is the initial velocity
a is the acceleration
v is the velocity at time t (t is a scalar)
r is the displacement at time t

2. To solve problems involving variable acceleration in 2 dimensions, use calculus with vectors by considering each function of time (the vector component) separately.

3. When integrating a vector for a variable acceleration problem, the constant of integration, c, will also be a vector.

4. To find constants of integration, look for initial conditions or boundary conditions.

5. Displacement, velocity & acceleration can be given using ***i*-*j*** notation, or as column vectors.

FORMULAE

The formula to find **the position vector, r,** of a particle starting at position $\boldsymbol{r}_0$ that is moving with constant velocity **v** is

$$\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$$

Constant acceleration vector equations:

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$\boldsymbol{r} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$

Calculus for variable acceleration:

Velocity, if displacement is a function of time:

$$\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t}$$

$$\int (\boldsymbol{v})\,\mathrm{d}t = \boldsymbol{s}$$

Acceleration, if velocity is a function of time

$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{s}}{\mathrm{d}t^2}$$

$$\int (\boldsymbol{a})\,\mathrm{d}t = \boldsymbol{v}$$

DOT NOTATION & DIFFERENTIATING VECTORS

Dot notation is a shorthand for differentiation with respect to time.

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} \qquad \dot{y} = \frac{\mathrm{d}y}{\mathrm{d}t} \qquad \ddot{x} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} \qquad \ddot{y} = \frac{d^2y}{dt^2}$$

To differentiate a vector quantitiy in the form f(t)i + g(t)j, differentiate each function of time separately.

If $r = x\mathrm{i} + yj$, then $v = \frac{\mathrm{d}r}{\mathrm{d}t} = \dot{r} = \dot{x}\mathrm{i} + \dot{y}j$ and $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2r}{\mathrm{d}t^2} = \ddot{r} = \ddot{x}\mathrm{i} + \ddot{y}j$

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